

SEQUENCE AND SERIES

✓ **Sequence** : A sequence can be regarded as a function whose domain is the set of natural numbers on some subset of it of the type $\{1, 2, 3, \dots, k\}$. Sometimes, we use the functional notation $a(n)$ for a_n .

✓ **Series** : Let $a_1, a_2, a_3, \dots, a_n$ be a given sequence. Then, the expression $a_1 + a_2 + a_3 + \dots + a_n + \dots$ is called the series associated with the given sequence.

$$a_1 + a_2 + a_3 + \dots + a_n = \sum_{k=1}^n a_k$$

✓ **Arithmetic progression** : $a, a+d, a+2d, \dots$

♥ **n^{th} term (General term)**

$$a_n = a + (n-1)d$$

$$l = a + (n-1)d$$

♥ **The sum of n terms**

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

♥ **Sum of A.P. when first and last term is given,**

$$S_n = \frac{n}{2} [a + l]$$

a = first term
 l = last term
 d = common difference
 n = the no. of terms
 S_n = the sum of n terms

✓ **Arithmetic Mean (A.M.)**

$$A = \frac{a+b}{2}$$

a and b = two numbers
 A = Arithmetic Mean

✓ **Geometric Progression (G.P.)**

$$a, ar, ar^2, ar^3, \dots$$

♥ **General term of G.P.**

$$a_n = ar^{n-1}$$

r = common ratio

♥ **Sum of n terms of G.P.**

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

Case I If $r = 1$

$$S_n = na$$

Case II If $r \neq 1$

$$S_n = \frac{a(1-r^n)}{1-r}$$

OR

$$S_n = \frac{a(r^n-1)}{r-1}$$

$\rightarrow r < 1$

$\rightarrow r > 1$

✓ **Geometric Mean (G.M.)**

$$G = \sqrt{ab}$$

$; a, b > 0$

✓ **Relationship between A.M and G.M.**

$$A - G = \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0$$

$; a, b > 0$

✓ **Sum of first n natural numbers**

$$S_n = 1 + 2 + 3 + \dots + n ; S_n = \frac{n(n+1)}{2}$$

✓ **Sum of squares of the first n natural numbers**

$$S_n = 1^2 + 2^2 + 3^2 + \dots + n^2 ; S_n = \frac{n(n+1)(2n+1)}{6}$$

✓ **Sum of cubes of the first n natural numbers**

$$S_n = 1^3 + 2^3 + 3^3 + \dots + n^3 ; S_n = \frac{[n(n+1)]^2}{4}$$